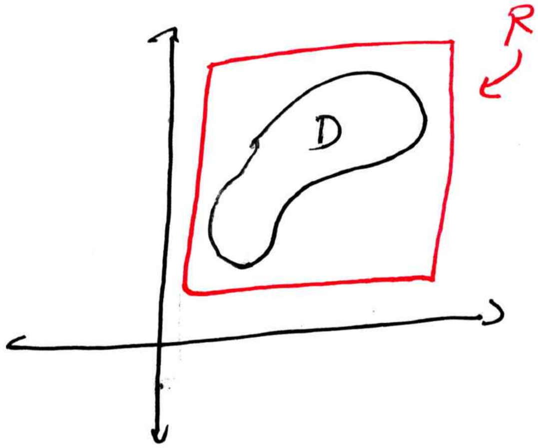
PROJECTED WRITTEN NOTES FROM THE M 408D(1-2) LECTURE ON MONDAY, NOVEMBER 23, 2020

Sec. 15.2 : Defining Double Integrals over more General Regions D.

Let $z = f(x,y)$ be given. Suppose D is a <u>BOUNDED</u> region in the x,y plane that is contained in the domain of f.



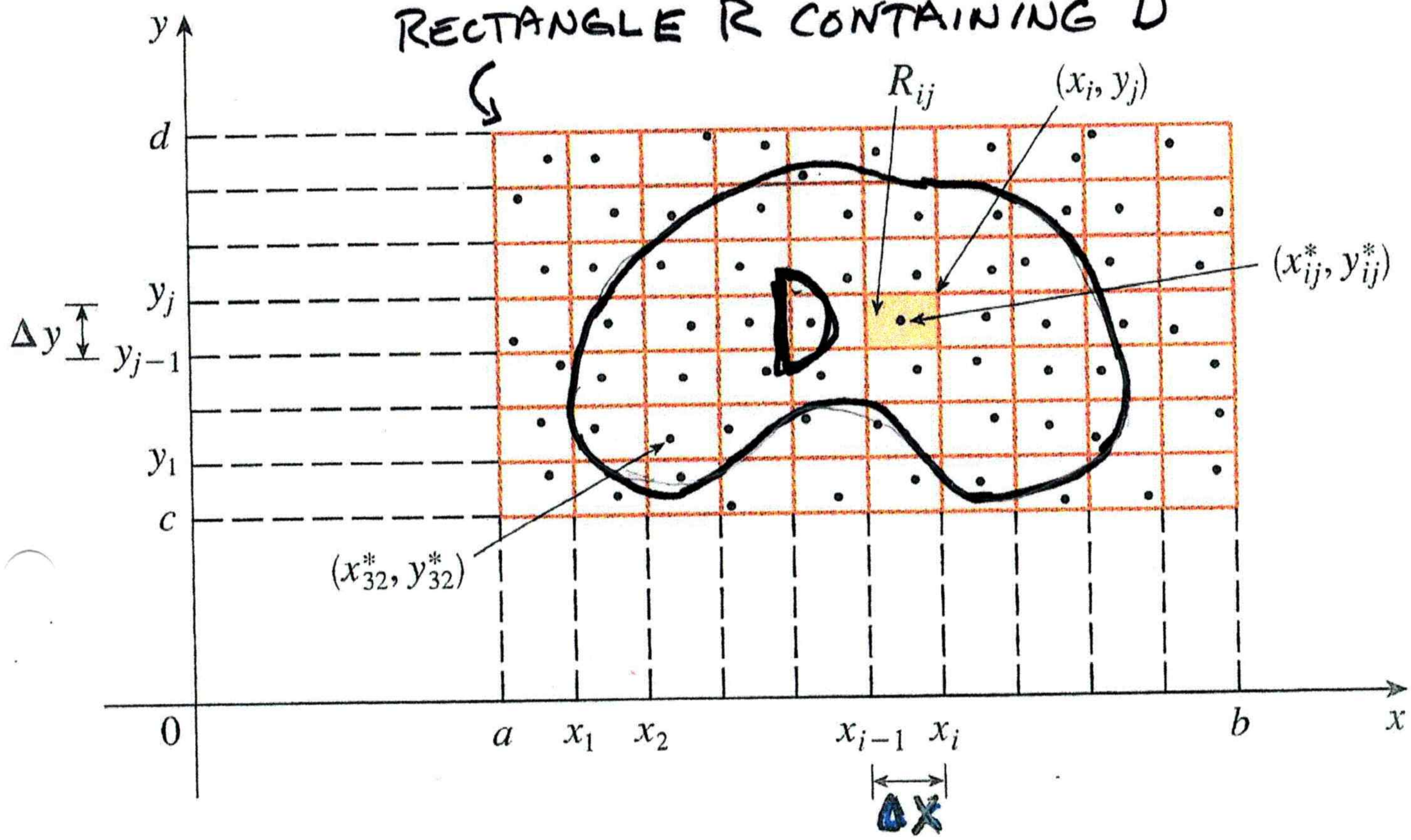
Dr. Shirley's Defn of the Double Integral region D.

$$\iint_D f(x,y)\,dA = \lim_{\substack{m \to \infty \\ n \to \infty}} \Big(\underbrace{\sum_{i's} \sum_{j's} f(x_{ij}^*, y_{ij}^*)\,\Delta A_{ij}}_{\text{Only for those selected points } (x_{ij}^*, y_{ij}^*) \text{ such that } (x_{ij}^*, y_{ij}^*) \text{ is in Region D.}} \Big)$$

How do we determine what number this is?

$$\iint_D f(x,y)\,dA = \lim_{\substack{m\to\infty \\ n\to\infty}} \Bigg(\underbrace{\sum_{i\text{'s}} \sum_{j\text{'s}}}_{\text{where the restrictions below hold.}} f(x^*_{ij}, y^*_{ij})\,\Delta A_{ij} \Bigg)$$

RECTANGLE R CONTAINING D



HERE, the Double Riemann Sum $R_{m,n}$

is $R_{m,n} = \underbrace{\sum_{i\text{'s}} \sum_{j\text{'s}}}_{} f(x^*_{ij}, y^*_{ij})\,\Delta A_{ij}$

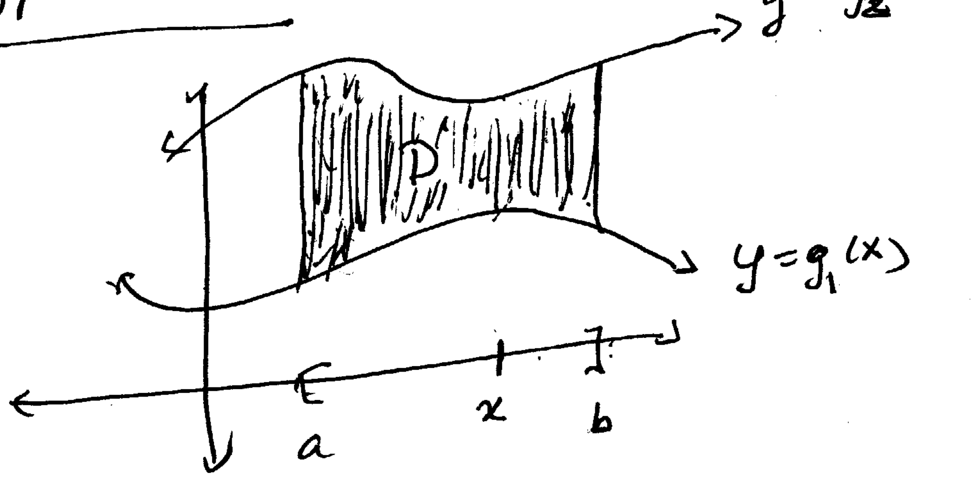
Only for those selected points (x^*_{ij}, y^*_{ij}) such that (x^*_{ij}, y^*_{ij}) is in Region D.

We can use iterated integrals when:

(1) If the Region D is a Type I Region,

(2) If the Region D is a Type II Region.

Type I REGION



For all $a \le x \le b$, $g_1(x) \le g_2(x)$

Type I Description of D:

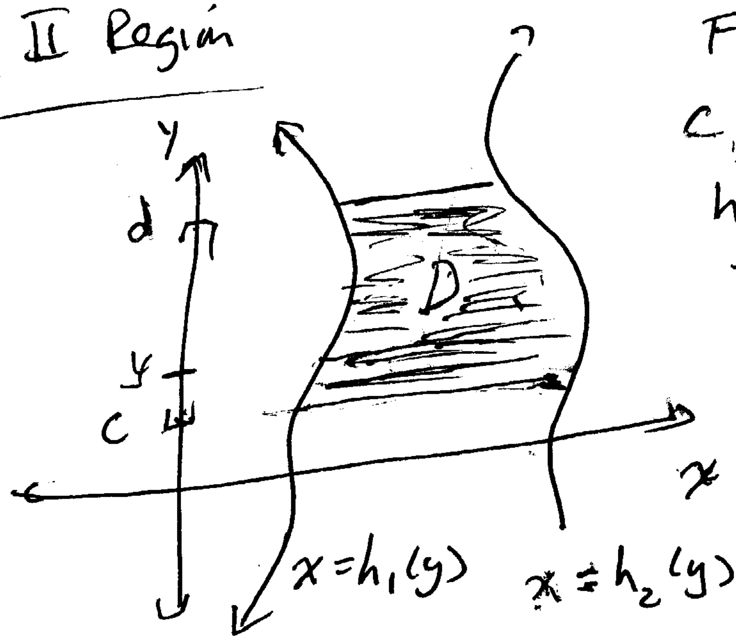
$$D = \{(x,y) \text{ such that } a \le x \le b, g_1(x) \le y \le g_2(x)\}$$

Shorthand: $D: a \le x \le b$ and $g_1(x) \le y \le g_2(x)$

When D is a Type I Region,

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

Type II Region



For all y
$c \le y \le d$,
$h_1(y) \le h_2(y)$

A Type II Description of D.

$D = \{ (x,y)$ such that $c \le y \le d,\ h_1(y) \le x \le h_2(y) \}$

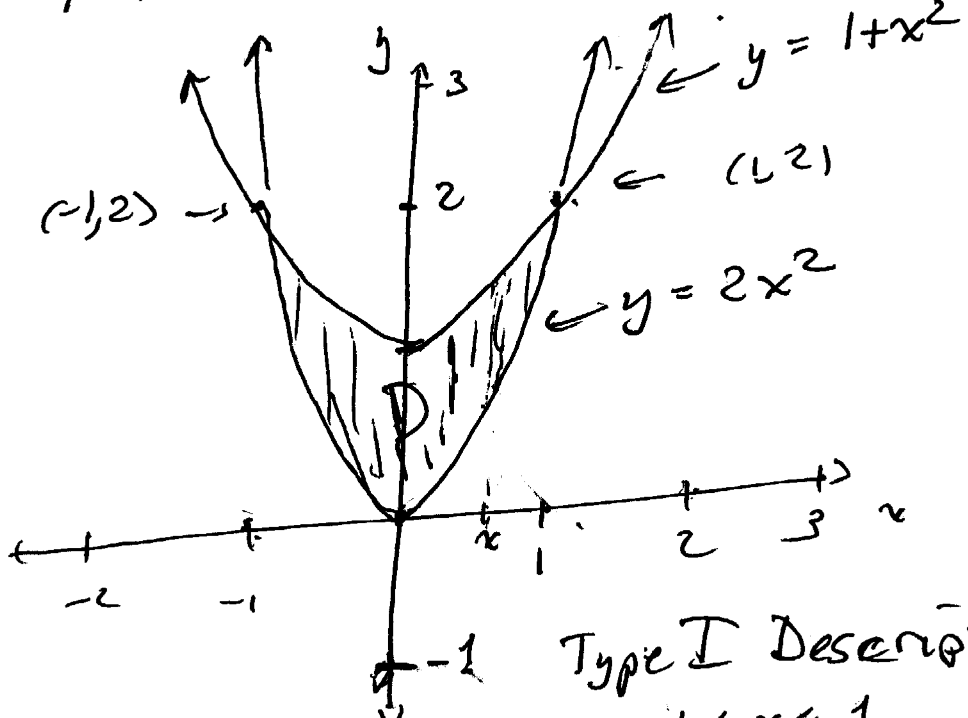
$D: c \le y \le d$ and $h_1(y) \le x \le h_2(y)$

When D is a Type II Region,

$$\iint_D f(x,y)\,dA = \int_c^d \underbrace{\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx}\,dy$$

Problem: Determine $\iint\limits_D (x+2y)\,dA$

where D is the Region bounded by
$y=2x^2$ and $y=1+x^2$.



Type I Description of D
$-1 \le x \le 1$
$2x^2 \le y \le 1+x^2$

$$\iint\limits_D (x+2y)\,dA = \int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx$$

$y=2x^2$

$$= \int_{-1}^{1}\left((xy+y^2)\Big|_{y=2x^2}^{y=1+x^2} \right) dx$$

$$= \int_{-1}^{1}\Big(\big(x(1+x^2)+(1+x^2)^2\big) - \big(x(2x^2)+(2x^2)^2\big)\Big)\,dx$$

$$= \int_{-1}^{1} (-3x^4 - x^3 + 2x^2 + x + 1)\,dx$$

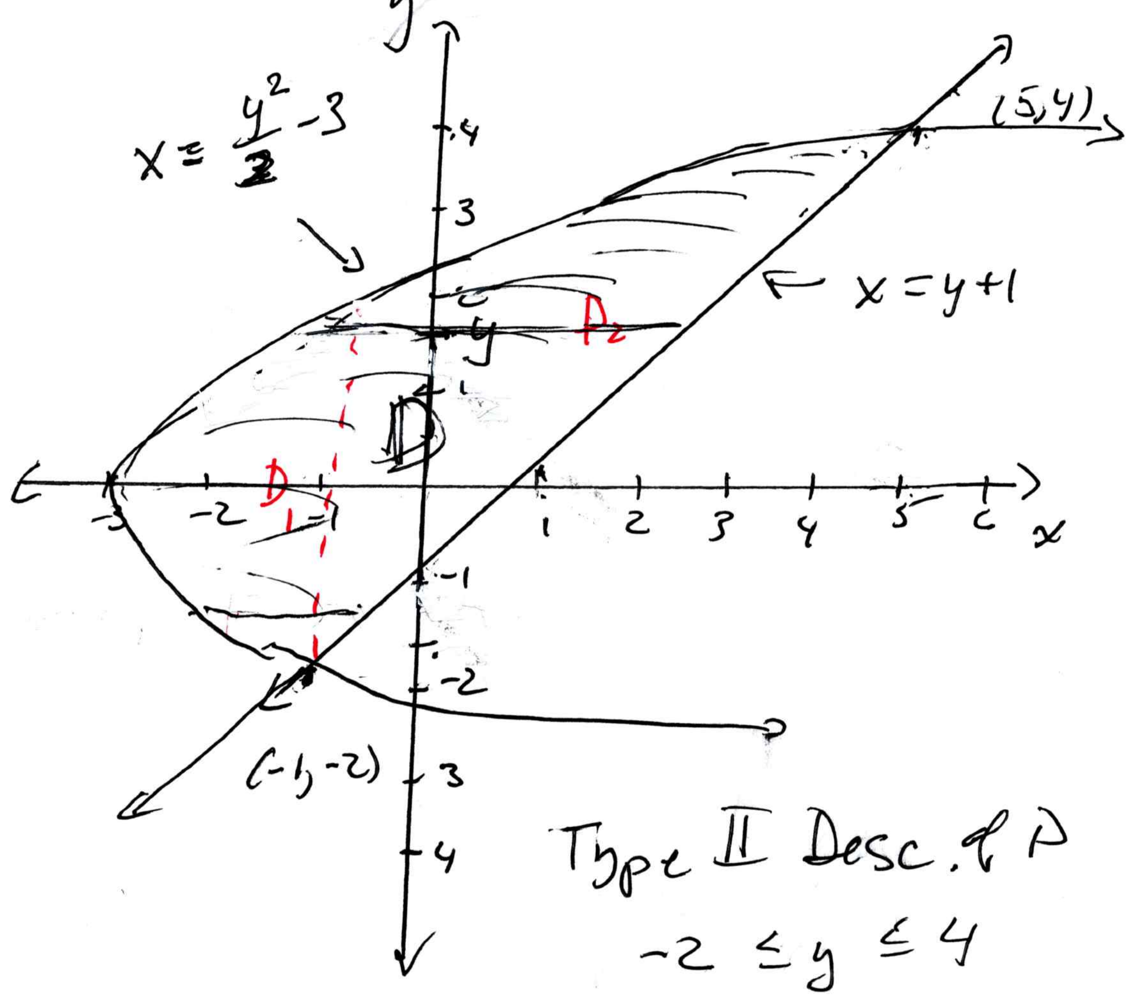
$$= \left(-\frac{3}{5}x^5 - \frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + x\right)\Big|_{-1}^{1}$$

$$= \cdots = \frac{32}{15} = 2\,{}^{2}\!/_{15}$$

$$\iint_D (x+2y)\,dA = \frac{32}{15} = 2\,{}^{2}\!/_{15}$$

A Type II Example

Find $\iint_D (xy)\,dA$ where D is the Region bounded by $x = \frac{y^2}{2} - 3$ and $x = y+1$



Type II Desc. of D

$$-2 \le y \le 4$$

$$\left(\frac{y^2}{2} - 3\right) \le x \le (y+1)$$

$$\iint_D (xy)\,dA = \int_{-2}^{4} \left(\int_{x=\frac{y^2}{2}-3}^{y+1} xy \, dx \right) dy$$

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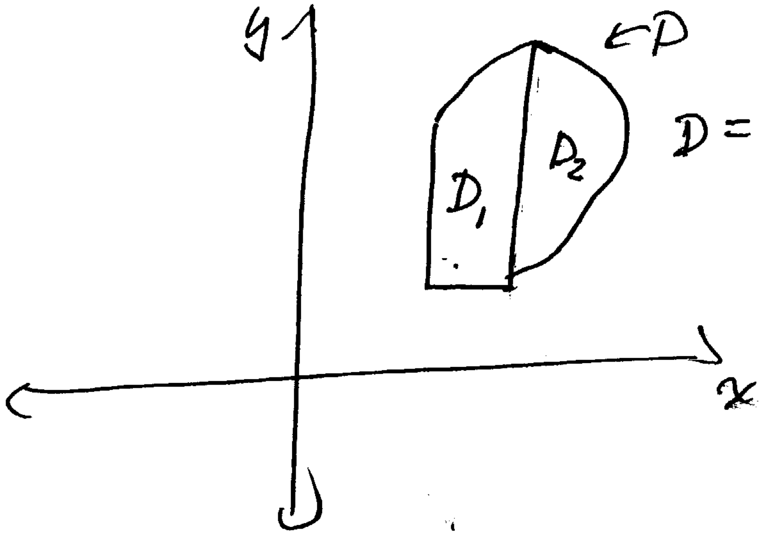
A Property of the Double Integral



$D = D_1 \cup D_2$

$$\iint_D f(x,y)\,dA = \iint_{D_1} f(x,y)\,dA + \iint_{D_2} f(x,y)\,dA$$